MATH 4123 HISTORY OF MATHEMATICS Problem Set II Name:
PROF. PAUL BAILEY November 15, 2006

Due Date: Wednesday, November 22, 2006.
Write your solutions neatly on separate pieces of paper and attach this sheet to the front.

Problem 1. Compute the area of a regular pentagon inscribed in a unit circle.

Problem 2. Consider the elliptic curve given by the equation

$$y^2 = x^3 - 12x + 25.$$

Find as many rational points on this curve as you can, including all rational points that lie on a horizontal tangent. Justify your answer.

Problem 3. Find the solution set of the polynomial equation

$$x^4 - x^3 - 3x^2 + 8x - 5 = 0.$$

Problem 4. Let $(F_n)_{n=1}^{\infty}$ be the Fibonacci sequence, and define a sequence $(b_n)_{n=1}^{\infty}$ by

$$b_n = \frac{F_{n+2}}{F_n}.$$

Find $\lim_{n\to\infty} b_n$.

Problem 5. Planet Minerva has an elliptical orbit with the sun at one vertex. Sixty percent of the time, its distance from the sun is greater than its semimajor axis. Use this information and Kepler's second law of planetary motion to compute the eccentricity of its orbit.